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# Relationships between teachers' background, their subject knowledge and pedagogic efficacy, and pupil achievement in primary school mathematics in Hong Kong; an indicative study

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Keywords

Hong Kong; mathematics subject matter knowledge; pedagogic efficacy; performance of mathematical operations; mathematics achievement

# Relationships between teachers' background, their subject knowledge and pedagogic efficacy, and pupil achievement in primary school mathematics in Hong Kong; an indicative study


## Abstract

This study investigates how teacher background, subject knowledge and pedagogic efficacy affect Grade 4 children's (aged 9-10) mathematical achievement in 10 primary schools in Hong Kong. Mathematics teachers were selected for their strong commitment to teaching mathematics and their pupils' consistently high international mathematics performance. Teacher measures (i.e. level of mathematics education, teaching experience, mathematics subject knowledge, performance in mathematical operations and pedagogic efficacy) were checked for factor consistency and assessed against age-appropriate pupil mathematical achievement. The results showed that teachers were secure in their subject knowledge, and that such knowledge was related to their performance of mathematical operations, but it was high levels of pedagogic efficacy and the ability to perform age-appropriate mathematics operations (rather than subject knowledge) that positively affected their pupils' achievement. These findings contradict ongoing international calls for the enhancement of primary school teachers' mathematical subject knowledge, as they show pedagogic efficacy to be more strongly associated with pupil achievement.



## Keywords

Hong Kong; mathematics subject matter knowledge; pedagogic efficacy; performance of mathematical operations; teacher background

## Introduction

Concerns about the relationship between teachers' mathematical subject knowledge and pupils' performance have been expressed internationally over the past four decades. These concerns raise questions of policy and practice, and their underlying premise is that greater subject knowledge amongst mathematics teachers will improve children's mathematical understanding and achievement (e.g. Office for Standards in Education [Ofsted], 2000; US Department of Education [US DoE], 2008). Worries over children's mathematics performance have been exacerbated by: 1) international comparisons of such performance that show the gradual decline of Western countries in relation to Asian countries (Henderson, 2012; Mullis, Martin, Fay & Arora, 2012; Organisation for Economic Cooperation and Development [OECD], 2010); 2) international curricular recommendations to enhance children's mathematical engagement and achievement by moving, for example, from the traditional transmission teaching approach to inquiry-oriented learning (Ball, 1988; National Council of Teachers of Mathematics [NCTM], 1989; Yates & Collins, 2010); and 3) the realisation that pupils' mathematics achievement may also be affected by their teachers' pedagogic efficacy and associated classroom actions (Ross, Hogaboam-Gray, & Gray, 2003; Stipek, Givven, Salmon, & MacGyvers, 2011).

In the quest to improve primary school children's mathematical engagement and achievement, one frequently cited assumption is that teachers' mathematics subject knowledge is key to such improvement (Aubrey, 1997; Ball, Hill, & Bass, 2005; Goulding, Rowland, & Barber, 2002; Ma, 1999; Ofsted, 1994, 2000; US Department of Education, 2008). Research has addressed the issue of teachers' subject knowledge through explorations of the level of mathematical training they received in their own secondary and tertiary education (Ball, Lubienski, & Mewborn, 2001; Henderson & Rodrigues, 2008; Hill, Rowan, & Ball, 2005), training to increase mathematics knowledge via pre-service courses and testing (Brown, McNamara, Jones, & Hanley, 1999; Henderson, 2012) and in-service training (Wilkins, 2008).

Yet, as Hill et al. (2005), Wilkins (2008) and others have recognised, there are no studies actually showing a direct relationship between teachers' mathematical subject knowledge and their pupils' mathematical achievement. The absence of such a relationship points to the need for further research and more sophisticated arguments concerning other teacher-based factors that may also promote children's mathematical achievement, such as amount of mathematical training, teaching experience, pedagogic efficacy and actions/interactions in the classroom (Ma, 1999).

## Background of the Study

The setting for this study was Hong Kong (HK), where mathematics teachers have been encouraged to move away from didactic teaching methods (Curriculum Development Council Hong Kong [CDCHK], 2001) and schoolchildren score consistently highly in mathematical understanding in international assessments (Mullis et al., 2012; OECD, 2010). Its data were taken from a larger-scale quasi-experimental project that investigated the promotion of children's mathematical understanding via an enhanced social pedagogic teaching intervention[1]. Five hundred and four pupils and twenty primary school mathematics specialist teachers in HK formed the sample. Each of the teachers involved had expressed interest in improving their classroom skills and volunteered to participate in the research (refer to *The Present Study* section below for further information on the teachers' background). Data were collected to elicit the teacher self-reported characteristics identified by Rowland, Martyn, Barber, and Heal (2001), including teachers' mathematics subject knowledge, level of mathematical training undertaken prior to becoming a teacher, number of years teaching

[1]The project was a collaborative effort between two research teams in the University of Cambridge and one of the universities in Hong Kong, which involved 13 academic scholars and research assistants in both places participating in a variety of research activities such as teacher training workshops and classroom observations. It was supported by the Economic and Social Research Council (UK) and Research Grants Council (HK) joint-funding scheme.

mathematics, performance confidence in undertaking age-appropriate mathematical operations and age-appropriate pedagogic efficacy (i.e. confidence in teaching mathematics topics associated with pupils' learning engagement). These data provided the criteria for assigning teachers/classes to experimental and control groups in the larger-scale project, and allowed us in the present study to assess whether pupil mathematics achievement in HK varies across classrooms and to seek explanations for such achievement based on a range of teacher characteristics.

## Theoretical Framework

Since the 1980s, educational policy in a number of countries has increasingly become concerned that primary school teachers' limited mathematical subject knowledge is responsible for their pupils' low levels of mathematical achievement (Ma, 1999). Concern over the teacher subject knowledge-pupil achievement link prompted Shulman (1987) to develop terminology for different types of self-reported teacher knowledge in the education context. Whilst not referring to mathematics per se, Shulman differentiated amongst subject matter knowledge (SMK), pedagogic content knowledge (PCK), curriculum knowledge (CK) and five other aspects of knowledge used for teaching. Drawing upon this vocabulary, a number of studies focusing on the mathematics curriculum have been undertaken in an attempt to link SMK (often supported by PCK) to pupils' mathematical achievement (Ball et al., 2005; Goulding et al., 2002; Ma, 1999; National Commission on Teaching and America's Future, 1996; Ofsted, 1994; Ubuz & Yayan, 2010; US DoE, 2008; Wong, Rowland, Chan, Cheung, & Han, 2008). Although a rather simplistic relationship between teacher SMK and pupil performance has been posited, the results of international studies have been ambiguous. For example, Askew, Rhodes, Brown, William, and Johnson (1997), Hill et al. (2005) and Ma (1999) failed to document a direct relationship between SMK and pupil achievement, although they noted that low levels of SMK are associated with poor pupil performance in mathematics. Further, Muijs and Reynolds (2002) suggested that a curvilinear relationship may exist between SMK and performance, with both low and high levels of SMK associated with poor pupil performance.

Studies investigating the issues of teachers' self-beliefs, pedagogic activity and classroom activity have further indicated just how problematic the proposed SMK-performance relationship is (Henderson, 2012; Hill et al., 2005; Muijs & Reynolds, 2002). Research shows that teachers who lack specific aspects of SMK may avoid teaching specific topics or may teach them using traditional/transmission techniques (Cunningham & Blankenship 1979; Goulding et al., 2002; Ofsted, 1994, 2000; Ramsey-Gassert & Scroyer, 1992). In sum, whilst SMK has been generally linked to pupil performance in certain school subjects (including mathematics), the relationship has often been weak or statistically insignificant (Carlisle, Correnti, Phelps, & Zeng, 2009; Moats, 1994), possibly because few studies have controlled for pedagogic efficacy or because a lack of CK is compensated for by effective teaching techniques (Harlen, Holroyd, & Byrne, 1995). In this respect, Shulman (1986) noted that "[m]ere content knowledge is likely to be useless pedagogically" (p. 8), and Goulding et al. (2002) hypothesised that teachers with low levels of SMK may spend more time preparing lesson plans. Both comments suggest that studies attempting to relate SMK to pupil achievement in mathematics should also take into account such teacher attributes as the amount and type of training and teaching experience, the classroom teaching context and a measure of pedagogic efficacy with regard to the subject (Goulding et al., 2002; Muijs & Reynolds, 2002). The failure to identify a conclusive relationship between SMK and achievement may also be explained in part by the use of small-scale, qualitative methodologies that do not account for the actions and interaction of the classroom and pupil contexts (Brown et al., 1999; Hill et al., 2005; Muijs & Reynolds, 2002; Wilkins, 2008).

The ambiguities identified concerning the relationship between SMK and achievement prompted Rowland, Huckstep, and Thwaites (2003) to expand theoretical and practical understanding of SMK into a 'knowledge quartet' emphasising that direct knowledge of a subject, the theoretical underpinnings of subject content, and pedagogical planning and techniques are all important elements in advancing pupils' mathematical achievement. They also identified a mediated relationship between SMK and achievement, noting that teachers' mathematical efficacy and classroom practice are associated with pupils' mathematics performance. These findings require an explanation of teacher efficacy, particularly with regard to the pedagogy likely to be used in the classroom, as both low and high levels of reported SMK have been associated with traditional transmission teaching and a decreased likelihood that teachers will adopt student-centred (or interactive inquiry-based) teaching approaches (Czerniak, 1990; Haney, Czerniak, & Lumpe, 1996)

## Pedagogic Efficacy and the Classroom

A number of studies have considered whether and how general levels of teacher SMK and performance confidence affect pupil achievement via efficacy, although few have taken place within the mathematics subject area. The importance of teachers' self-reported pedagogic efficacy is highlighted by its definition: "a teacher's expectation that he or she will be able to bring about student learning" (Ross et al., 2003, p. 3). Underlying this definition are two distinct theoretical approaches that consider the importance of efficacy at a general level for teachers. From Ajzen (2002) comes the explanation that efficacy is the 'self-perception of control', which is likely to affect behaviour. Alternatively, Bandura (1997) explained efficacy as the belief in one's ability "to organize and execute the courses of actions required to produce given attainments" (p. 3). The combination of behavioural control and self-belief to effect actions has been identified by Lucas and Barge (2010) as key to the "prediction of professional intention and behaviour" (p. 505) and has been substantiated in meta-analyses of the literature (Lucas & Barge, 2010; Stajkovic & Luthans, 1998). Bandura (1997) qualified the conceptualisation of efficacy, noting that it is likely to be domain- (or subject-/task-) specific, and thus that the measurement of efficacy should assess the individual's confidence in performing specific domain-related tasks.

Teachers with a high degree of pedagogic efficacy demonstrate higher levels of pupil interaction than low-efficacy teachers (Czerniak, 1990; Smylie, 1988) and are more likely to attempt new teaching strategies (Haney et al., 1996) and enhance pupil achievement (Wilkins, 2008). Whilst a lack of subject confidence in teachers may be linked to generally poor pupil performance, Stipek et al. (2011) found that teachers' self-confidence, identified as pedagogic efficacy, is correlated with pupils' confidence as mathematics learners in what Henderson (2012) calls a 'competence/confidence spiral'. However, teachers' pedagogic efficacy and confidence concern more than a series of self-beliefs. They also encompass within-classroom pedagogic activity in the generation of confidence and performance related to the specific curriculum tasks that teachers prepare and present to their pupils (Muijs & Reynolds, 2002; Henderson, 2012; Cobb & McClain, 2006). Programmes that enhance teachers' pedagogic efficacy rather than focusing on CK may be able to mitigate the negative effects of a less extensive background in mathematics (Wilkins, 2008).

With regard to pupils' learning and achievement in mathematics, it is likely that teachers will need to demonstrate domain-specific mathematics pedagogic efficacy (Bandura, 1997). A number of studies have adapted general teacher efficacy scales for use with mathematics teachers (see especially Enochs, Smith & Huinker, 2000). For example, the Mathematics Teaching Efficacy Belief Instrument (MTEBI) developed by Enoch et al. (2000) collects information from mathematics teachers on elements of control (Rotter, 1966) and expected pupil learning outcomes (Bandura, 1997). Research using the MTEBI has shown greater participation in mathematics pedagogic methods courses to be associated with higher levels of reported mathematics teaching efficacy (Cakirolu, 2000; Wenta, 2000). However, the MTEBI and similar instruments have been used only to assess the effects of pre-service training courses on mathematics teachers' efficacy. They do not assess how in-service mathematics teachers' pedagogic efficacy affects their pupils' mathematics achievement. Hence, a more appropriate instrument needs to be developed to clearly link teachers' curricular efficacy to their confidence in teaching appropriate subject matter and to actual pupil achievement in mathematics.

## Research Problem

The inter-relationships amongst teachers' training and teaching experience, mathematical SMK, ability to perform mathematically, pedagogic efficacy and pupil performance discussed in the foregoing section are well suited to investigate in HK for several reasons. For example, a study of HK teachers' SMK would be interesting because of HK students' strong performance on various international mathematics assessments (e.g. Trends in International Mathematics and Science Study [TIMSS] and the Programme for International Student Assessment [PISA]), particularly in light of the HK education authority's recommendation that mathematics teachers become less didactic and encourage greater pupil engagement in classroom discussion and argumentation (CDCHK, 2000). Wong et al. (2008) noted that pre-service and practising mathematics teachers in HK have relatively high levels of SMK, although some hold a number of mathematical misconceptions. Tsang and Rowland (2005) reported a contradictory finding: although they found HK teachers' SMK to be equivalent to that of teachers in the UK, students in the UK score much lower in the TIMSS and PISA tests. Schoon and Boon (1998) hypothesised that SMK misconceptions may lead to reduced pedagogic efficacy. There is also a gap in the literature on teachers' pedagogic efficacy as an intervening variable between teacher knowledge, certification, and experience/background and pupil performance. Filling that gap requires a study that ascertains whether pupil achievement is affected by teachers' background, knowledge and pedagogic factors, as well as the effects of classroom interaction. Accordingly, the current study, which took place in a high-performing (in mathematics) region, explored the relative effects of teachers' subject knowledge, confidence in performing subject-based tasks, pedagogic efficacy in subject teaching and personal mathematics background on pupils' mathematical performance. It was guided by the following research questions.

1. Are teachers' level of mathematical training, teaching experience, SMK, performance competence and pedagogic efficacy related?
2. Do any of those five factors affect pupils' mathematical achievement?
3. Which of those factors, i.e. mathematical training, teaching experience, SMK, performance competence and pedagogic efficacy, or combination thereof is most likely to explain pupils' mathematical achievement?

## The Present Study

### I) Methods

The data for this study were collected in a one-time survey of pupils' mathematics achievement and characteristics of their mathematics teachers in HK primary schools. The teachers came to the attention of the researchers via a university/school teacher training partnership. Each of the participating teachers was a mathematics specialist in her/his school; which characterises mathematics teaching in upper primary schools in HK. Being a specialist teacher meant that their predominant teaching responsibility was the mathematics curriculum, although some of the teachers taught subsidiary subjects as well. The teachers involved expressed an interest in the further development of their classroom pedagogic skills before they agreed to join the project. Teachers were recruited from the span of schools with different 'bandings' (i.e. based upon perceptions of pupil academic ability) to assure a decent degree of variation of participants (i.e. teachers and pupils) for investigation. Upon gaining ethical permission to undertake the study, consent to participate was agreed at the school level (school principal), participating teacher level and classroom level via combined pupil and parental consent (including the right to withdraw). Data collection took place between late December 2012 and early January 2013. The timing of the survey afforded the teachers and pupils a full school term (from September to December) to achieve interpersonal, subject and pedagogic familiarity. To allow for comparability of the teachers' subject knowledge and pedagogic skill, the study focused on Primary 4 teachers and their classes only. The pupils were aged between 9 and 10, by which age they should have developed basic mathematics understanding (addition, subtraction, multiplication, division) and were not yet under pressure to prepare for their primary school leaving examination (which takes place in P6). All schools and classes were co-educational.

**II) Sample**

*a) Pupils:* The pupil sample comprised mixed-ability P4 classes taught by 19 of the participating teachers and one P4 special needs mathematics class, numbering 504 pupils with a 60:40 bias in favour of boys. The average class size was 22 pupils, indicating that most of the P4 classes had benefitted from the HK Education Bureau's decision to reduce primary school class sizes to below 25 (see Galton & Pell, 2010).

*b) Teachers:* Twenty primary school mathematics teachers (10 male, 10 female) volunteered to participate. Most had studied mathematics to the Bachelor's degree level, with only two having ended their study of mathematics before completing their secondary education (see Table 1). All but one teacher had multiple years of mathematics teaching at the primary level, and the average was 10.5 years (standard deviation [SD] = 7.6). Nineteen of the teachers taught mixed-ability classes, and one teacher a small special needs class of 8 pupils. All teachers were bilingual in Cantonese and English.

[INSERT TABLE 1 HERE]

**III) Instruments**

*a) Pupils:* Pupils' mathematics performance was measured via a government-designed, 38-item test of mathematics problems covered in the prior P3 year (see samples in Appendix I). The question items asked pupils to solve mathematics problems involving simple and complex addition and multiplication, number order, geometry and charts and to show their calculations. Each item was scored on a 5-point scale: '0' [incorrect answer with no evidence of an attempt to work it out], '1' [either a correct answer with incorrect calculations or an incorrect answer with correct calculations], '4' [correct answer with correct procedure used to work it out]. The participating pupils underwent this assessment in late January 2013, five months into the school year.

*b) Teachers:* The Mathematics Subject Matter Knowledge Survey completed by the teachers was originally developed by Rowland et al. (2001). It provides self-reported age-appropriate measures of teachers' mathematical background (maths level), mathematical SMK, and mathematical performance confidence (PFC) and an associated scale of pedagogic efficacy (PEf) for teaching primary school mathematics[1] (see examples in Appendix II). The survey has been adapted and validated for use in Hong Kong (Tsang & Rowland, 2005; Wong et al., 2008). The following describes the instrument measures.

1. Mathematics level (Maths level) identified when teachers completed their study of mathematics on a 3-point scale: '1' [studied mathematics until age 16]; '2' [studied mathematics until age 18 (end of secondary schooling)]; '3' [studied mathematics at university].
2. Years of teaching experience – actual number of years teachers had taught mathematics in primary schools.

3. The SMK measure required teachers to complete 16 mathematical problems. Each answer was assessed on a 5-point scale ranging from '0' [not attempted or no solution] to '4' [completely secure in knowledge, with convincing and rigorous explanations]. All 16 problems have been assessed within the HK primary school mathematics subject domain, and cover "themes of basic arithmetic competence, mathematical exploration and justification, and geometric knowledge" (Tsang & Rowland, p. 4). These three themes were chosen because they are basic elements of the HK mathematics curriculum and address both substantive and syntactic knowledge of mathematics.

4. The PFC measure asked the teachers to describe their level of confidence in undertaking each of the SMK problems on a 5-point scale ranging from '0' [terrifying; I can't think about it] to '4' [I could explain to someone else how to solve this]. PFC was originally referred to as 'Self Audit' by Tsang and Rowland (2005), which was defined as teachers' "self-assessment of their confidence in successfully solving each item in the [SMK] Questionnaire before they actually started attempting that item" (p. 12).

5. The PEf scale asked teachers to rate their ability to teach 19 age-/curriculum-appropriate mathematics topics to their present class on a 5-point scale ranging from '1' [I hate teaching this topic, and the pupils find it difficult] to '5' [I love teaching this topic, and the pupils have fun with it]. The PEf ratings contain intertwined elements of teachers' personal teaching efficacy (Rotter, 1966) and "beliefs in one's capabilities to organize and execute the courses of action required to produce given [pupil] attainments" (Bandura, 1997, p. 3). The presentation of the PEf scale in this study differed from previous mathematics pedagogic efficacy ratings in that the PEf topics focused on actual mathematics topics in the P4 curriculum rather than on more general questions concerning the role of mathematics teaching. The current PEf scale may also assess applied PCK (Shulman, 1987) to a certain extent, albeit not formally, as it draws on

an age-appropriate, domain-specific combination of teaching approaches and subject knowledge used in the P4 mathematics curriculum.

**IV) Data analysis:**

Given the instruments used, sample size and classroom nesting involved in this study, a number of analytic strategies were employed, as follows.

*a) Re-establishing the ecological and content validity of the SMK Audit and PEf items*: As noted, the SMK, PFC and PEf scales had previously been validated in HK (Tsang & Rowland, 2005), but that validation took place over 10 years ago. Also, for the SMK scale, the original validation focused on just 10 of the 16 items. After discussions with non-sample teachers and teacher trainers, it was decided that teaching practices may have evolved over the past 10 years, and thus that checks for ecological and content validity needed to be repeated (particularly with regard to the full 16-item SMK scale). Ecological validity was assessed via interviews with non-sample P4 and P5 teachers, reviewing each survey question to ensure that it was relevant to and answerable by P4 teachers in either English or Cantonese. All questions were found to be answerable and to have content relevant to the P4 mathematics curriculum. None required rewording.

Content validity was assessed using confirmatory factor analysis (CFA; Hoyle, 2000) performed separately for each of the three scales, with varimax rotation, a minimum Eigen value of 1.0 and a minimal loading threshold of 0.5 (Tabachnick & Fidell, 2013). Upon meeting each of the minimum CFA requirements, the reliability of each scale was assessed (McMillan & Schumacher, 2001) using the 'alpha-if-deleted' test. Minimum expectations for reliability of 0.7 were surpassed for the SMK, PFC and PEf scales (see results below).

*b) Stepwise regression and CART:* These strategies were employed to identify a relevant mediational analysis of pupil achievement outcomes within a limited sample size and classroom-clustered context. Given that data were collected from pupils and their teachers, allowing an assessment of pupil achievement and teaching background, we were particularly concerned with whether outcomes were mediated by SMK, PFC and/or PEf (Krull & MacKinnon, 2001; Bryk & Raudenbush, 1992). One recommended way of handling outcome data from individual pupils who are also clustered at the classroom level is multilevel modelling (MLM). However, our sample size fell below the '30/30' rule (Kreft, 1996, cited in Hox, 2007), meaning that the accuracy of MLM could not be guaranteed. In preference to MLM, we decided to adopt an exploratory strategy to determine the relationship between the main teacher variables and pupils' mathematical achievement via stepwise regression and to initiate an explanation of the clustered results via a Classification and Regression Tree (CART).[2] Stepwise regression, whilst criticised as an analytic method (Whittingham, Stephens, Bradbury, & Freckleton, 2006), is useful for identifying which of the independent variables in a study affect a given dependent variable (pupil achievement in our case). Stepwise regression allows differentiation between independent variables likely and unlikely to contribute to an outcome while prioritising the variance explained by particular independent variables. If used only as an indicator, rather than as an assessment of true causality in relation to the dependent variable, stepwise regression prioritises the types of causal explanations that may be applied to the data. In other words, stepwise regression can only be considered indicative.

The CART method was also drawn upon to provide a fuller explanation for the effects of teachers' background and scale results on pupil achievement. CART is a statistical technique that provides a 'data mining' approach "to identify underlying factors that influence the dependent variable by consecutively clustering various cases into mutually exclusive groups that have noticeably distinctive values" (Walker, Lee, & Bryant, 2014, p. 610). It estimates the effects of independent variables on a dependent variable without requiring a specific model to work from (Ma, 2006), which allows the identification of the main variables and their interactions that are associated with the dependent variable (Walker et al., 2014). The CART approach is applicable to both parametric and non-parametric data. Whilst CARTs are useful for identifying the relationships between variables, caution must be exercised to ensure that the analyses do not become overly complex and provide results with limited generalisability (Bramer, 2007).

## Results

The results are initially presented at the descriptive level, followed by greater analysis in the discussion of the stepwise regression and CART results.

### I) Confirmatory factor analysis:

For the SMK scale, 12 of the 16 original problems met the minimal loading threshold (Tabachnick & Fidell, 2013) and provided a single factor. These problems accounted for 33.44% of the variance in the scale, with a high degree of reliability ($\alpha$ = 0.87). For the PFC scale, teachers' reflections on all 16 problems met the minimal loading threshold and combined into one factor, accounting for 49.16% of the variance with a high degree of reliability ($\alpha$ = 0.92). Finally, for the PEf scale, 17 of the 19 original problems met the minimal loading threshold and provided one single factor, accounting for 54.24% of the variance with a high degree of reliability ($\alpha$ = 0.96).

### II) Initial descriptive analysis:

Descriptive means were calculated from the confirmed teacher scales, teachers' level of mathematical education and amount of teaching experience and pupils' level of mathematical achievement (see Table 1). The results for the SMK and PFC scales showed that, on average, the teachers were 'secure in part/insecure in part' and 'confident in solving the problems', respectively, whereas those for the PEf scale revealed their perceived ability to teach the topics as 'good' (rather than excellent) and that most pupils liked studying the topics. The survey items on teachers' level of mathematics study showed that all 20 teachers had studied the subject throughout their secondary schooling and that more than half had completed an undergraduate degree with a mathematics major.

Initial analysis of the variance in pupil achievement by teacher revealed a significant difference in such achievement across the 20 classes ($F$[19, 476] = 9.88, $p < 0.001$, eta = 0.28). Even after excluding the special needs class, there was still a significant cross-class difference in mathematics achievement ($F$[18, 469] = 8.01, $p < 0.001$, eta = 0.24). In light of these large achievement differences amongst the classes/teachers, further descriptive analyses focused on formulating initial explanations with reference to teachers' background details and scale scores. To gain insight into the possible inter-relationships between the scales and background variables, correlation analysis was performed (Table 2). The Pearson correlations were not particularly strong, showing only SMK to be positively related to PFC, with all other correlations non-significant. It is worthy of note that, aside from the SMK scale, the other scales were negatively related to the background variables of mathematics level and years of teaching experience.

To ascertain whether any of the individual teacher factors affected pupil achievement, initial regressions and t-tests were carried out between the pupils' mathematical achievement and the scales/background variables (Table 3). The findings showed no significant differences between pupils' scores and SMK or PFC, but significant differences between those scores and PEf, mathematics level and teaching experience. Thus, while neither SMK nor PFC affected pupil achievement, higher teacher PEf scores equated to higher pupil achievement scores. More years of mathematics study by teachers was also found to be related to better pupil achievement. The negative t-score for the teaching experience/pupil achievement regression indicated that younger teachers affected pupil achievement more positively than their counterparts with more experience.

[INSERT TABLES 2 AND 3 HERE]

**III) Stepwise regression and CART:**

Given that the foregoing descriptive results ran counter to the expectations in the literature and that the participating HK primary school mathematics teachers appeared to have high levels of PEf and a strong mathematics background, stepwise regression was performed to ascertain the relative contribution of each teacher factor to pupils' mathematical achievement scores. The results showed three factors to make a significant contribution to the variance: Teachers' PEf (Beta = 0.23) contributed the most, followed by mathematics background (Beta = 0.20) and, to a very limited extent, SMK (Beta = 0.09).

Whilst the stepwise regression indicated that pupil achievement was most likely affected by teachers' pedagogic efficacy, the statistics offered little insight into or explanation of how or why the various scales and background factors did or did not affect such achievement. Hence, a CART was created to shed light on how various teacher factors affected pupil performance at the classroom level (see Figure 1). The CART included all three scales, and teaching experience and level of mathematics studies were included as contributing factors to pupil achievement. When reading the figure, it should be noted that the bottom row of box plots identifies nested classroom levels of pupils' mathematical achievement, with the lowest-achieving classes on the left and highest-achieving classes on the right. Observation of the figure indicates that teachers' number of years of teaching experience did not make a significant contribution to pupil achievement, whereas their PEf level did. A moderate level (above 50 on the 95-point scale) equated to higher levels of pupil achievement. Correspondingly, teachers with a low PEf score (below 50 or one SD below the mean) had average levels of SMK, and the combination of low PEf/SMK was associated with the lowest levels of pupil achievement.

Interestingly, the left side of the figure indicates that even when teachers had low levels of SMK, pupil achievement was enhanced by moderate levels of PEf. The right side of the figure indicates that teachers with the highest levels of PEf taught the classes in which pupil achievement was highest (two SDs above the mean), and there was little variation in pupil achievement in these classes (see distributions in the box plots). The classes whose teachers had moderate levels of PEf were also affected by those teachers' PFC. In these cases, relatively high levels of PEf and moderate-to-low levels of PFC were associated with high levels of pupil achievement, whereas moderate levels of pupil achievement appear to be explained by average levels of PEf and moderately high levels of PFC (one SD above the mean). Only in the lower levels of PEf can SMK be integrated into the explanation. Here, average levels of PEf combined with moderate levels of PFC/SMK appear to be associated with less mathematics study. The CART analysis bears some similarities to the stepwise regression results regarding the main contribution of PEf and limited contribution of SMK. It also adds additional detail to our explanation of nested pupil achievement at the classroom level, confirming the overall importance of teachers' PEf. Finally, although teachers' PFC was not significant in the regression, moderate scores on this scale were associated with higher levels of pupil achievement. In neither the stepwise regression nor CART did years of teaching experience make a significant contribution.

[INSERT FIGURE 1 HERE]

## Summary and Discussion

With regard to the three research questions, analyses of the data reveal the following.

1. Teachers' background details and scale scores (SMK, PFC and PEf) were not strongly related. There was clear correspondence only between teachers' SMK scores and degree of confidence in performing age-appropriate mathematics operations (PFC). A number

of somewhat surprising findings may be a quirk of the relatively high level of pre-service mathematics study in this particular sample: a) a strong mathematics background was only moderately (and not significantly) related to SMK, perhaps indicating a disjunction between mathematics education for pre-service teachers and the mathematics that they are likely to teach in their classrooms; b) all of the scales were negatively related to teaching experience, with the teachers with low-to-moderate levels of such experience more likely to have higher scale scores; and c) there was a positive correlation between PEf and the other scales, but the relationship between teachers' confidence in their ability to teach age-appropriate topics was not significantly related to their age-appropriate SMK or perceived competence in solving age-appropriate mathematics problems (PFC).

2. Pupils' mathematics achievement outcomes were not affected by teachers' SMK or PFC, but were strongly influenced by their PEf: teachers with the highest levels of PEf taught the classes with best outcomes. Also, although teachers' level of mathematics education and PEf were not related, each had a significant effect on the pupil outcomes. With regard to teaching experience, only low-to-moderate levels had a positive such effect.

3. Stepwise regression and CART analysis clearly showed high levels of teacher PEf to equate to high levels of pupil mathematics achievement. Whilst there was a clear relationship between PEf level and pupil outcomes, slightly lower levels of PEf were mediated by teachers' competence in performing mathematical operations (PFC) with regard to moderate pupil outcomes. Low levels of PEf and SMK were strongly associated with poor pupil outcomes.

The findings of this study carried out in a mathematically high-performing region such as HK call into question certain claims made in the literature and suggest that new factors need be considered in the provision of teacher training and professional development to primary school mathematics teachers. Governments around the world continue to state that these teachers need to enhance their SMK, but our findings suggest that such enhancement might not be effective in HK or other regions. As Rowland et al. (2001) note, knowing (SMK), doing (PFC) and having studied mathematics to degree level are only partially related to the effective teaching of mathematics. Enhanced pupil achievement is also related to teachers' PEf within the classroom, which is likely to be related to teachers' willingness to engage in mathematical discussions or interactions with pupils and to encourage discussion between pupils. These aspects of teachers' efficacious involvement with their mathematics pupils have become recommended practice worldwide (Ball, 1988; NCTM, 1989; CDCHK, 2000), and our research suggests they may be more important than enhanced SMK, which is associated with traditional teaching practices (Cunningham & Blankenship, 1979; Goulding et al., 2002; Ofsted, 2000).

Descriptive analyses provided an early indication of the limited effect of SMK on pupils' mathematical achievement. Although they showed a strong correlation between teachers' SMK and PFC, neither was related to pupil achievement. These findings appear to support Muijs and Reynolds' (2002) proposed 'threshold' effect for SMK, that is, a minimum level of subject knowledge is required to effectively teach mathematics. CART analysis also revealed that low SMK scores were associated only with the worst performing classes. High SMK levels appeared to exert no effect on pupil achievement. The lack of a significant correlation between PEf and SMK/PFC thus indicates that, for the group of teachers involved in this study, knowledge and competence in mathematics are separate from pedagogic considerations. In addition, neither SMK nor PFC was significantly related to teachers' mathematics background and amount of teacher experience, reinforcing the claim that the issues of pedagogy and pedagogic efficacy are distinct from simply knowing and doing mathematics for mathematics teachers (Ross et al., 2003).

PEf contributed most of the variance (over 23%) in pupils' mathematical achievement, suggesting that it is more than the 'mediating' factor posited by Ernest (1989). Our findings on PEf confirm Wilkins' (2008) assertion that teachers' (self) beliefs affect their classroom practice. In explaining why teachers' SMK does not affect pupils' performance, Wilkins (2008) noted that the effects of PEf are likely to be associated with higher levels of teacher-pupil interaction and lower levels of transmission-based teaching. That observation suggests that combining enhanced pedagogic efficacy with an inquiry-based/open teaching style could promote pupils' mathematics achievement, a conclusion that this study can only partially support as we were unable to assess teaching style or classroom interactions.

Finally, previous claims about the need for primary school mathematics teachers to obtain higher levels of mathematics education (particularly in relation to SMK; see Ubuz & Yayan, 2010; Norton, 2012) were only partially backed up by the results of this study. Teachers' mathematical background was not significantly correlated with any of the scales (i.e. SMK, PFC or PEf), although we did find a positive relationship between their level of mathematics study and SMK, perhaps indicating that general mathematics education is a more important factor than SMK in enhancing pupil achievement, particularly when combined with PEf (see Carre & Ernest, 1993).

The more complex and comparative regression and CART analyses we carried out further support the importance of pedagogic efficacy and its association with children's mathematical understanding. Historically, studies relating teachers' SMK to pupils' achievement have found little consistent support for a simple, direct relationship between the two variables, leading some researchers to highlight the greater importance of what happens in the classroom as opposed to what teachers bring to the classroom. Our PEf scale incorporated actual classroom topics and teachers' perceptions of their own teaching of and pupils' learning of those topics, and teachers' degree of mathematics efficacy was found to affect pupil achievement. More general studies of teacher efficacy suggest that teachers with high degrees of PEf are less didactic and more open to interactive teaching approaches. Wilkins (2008) further suggested that what happens in classroom-based teacher-pupil interactions is more 'telling' than the inconsistent results of SMK studies suggest. Acknowledging the transition in various mathematics curricula from transmission- to inquiry-based teaching approaches, Henderson (2012) posited that teachers need to play a more constructive role, and Cobb and McClain (2006) that pedagogic interaction and activity are central to children's learning. This more active, socially constructive role of teachers is perhaps captured in teachers' PEf scores in this study, which reflected both teacher and pupil interest and engagement in various mathematics topics (Askew et al., 1887; Muijs & Reynolds, 2002).

At the same time, we cannot disregard previous research concerning the (possible) SMK-achievement relationship. Our results may need to be qualified, as the study was undertaken with a small sample in a setting known for high levels of mathematics achievement. The participating teachers had also been involved in moves towards smaller primary class sizes and in the adoption of an inquiry-oriented approach to mathematics teaching, both of which have been found to facilitate interactive teaching and learning (Galton & Pell, 2010; CDCHK, 2000). We did find that a minimum amount of SMK appears to be necessary to arrest poor pupil performance, in concurrence with Muijs and Reynolds (2002). CART analysis further showed that a low level of SMK can be enhanced by high levels of PEf, and the relationship between them appears to enhance pupil achievement, at least when children are achieving at a moderate level. The most forceful result of this study is its identification of a strong association between high levels of PEf in teachers and high levels of pupil achievement, an association that was stronger amongst the teachers who had studied mathematics to the end of secondary schooling. Perhaps a solution to achieving and maintaining strong pupil mathematical performance in primary schools is to ensure that potential teachers continue their mathematics education to the end of secondary schooling and are provided with ample mathematics/teaching training opportunities to boost their pedagogic confidence. From the pedagogic and theoretical standpoints, our results provide insight into how primary school pupils can be taught mathematics more effectively. Importantly, they contradict calls to enhance primary school teachers' SMK, as they reveal no close relationship between such enhancement and pupils' mathematics performance. Rather, more time and exploration need to be devoted to boosting teachers' mathematics pedagogic efficacy, which is likely to be associated with in-class actions and interactions. Finally, although the sample size of the current research is relatively small compared to other international large-scale studies, it is argued that the strategic selection of teachers and pupils comprised an approximately representative sample of teachers (by background, experience and training), schools (by banded level and class size) and pupils (by previous levels of mathematics achievement) for Hong Kong. In this case, the interpretations of the results in the current study presented an insightful case with a (practically) limited sample size which duly informs the importance of teachers' pedagogic efficacy in pupils' mathematics achievement.

## Limitations and Future Research Directions

As noted at the start of this paper, our results are drawn from a relatively small sample of teachers with strong mathematics backgrounds. However, the results help to support previous criticisms of unproblematically accepting the linkage between teacher SMK and pupil achievement. We have moved away from qualitative studies (see Wilkins, 2008; Hill et al., 2005), adopting quantitative methods that allow a larger number of variables (Muijs & Reynolds, 2002) and exploration of the relative weighting (variance and interactions) between variables. Whilst we were unable to incorporate classroom observations into the study, our PEf measure was able to account in part for teachers' beliefs and their perception of classroom actions and interactions with their pupils as they pertained to confidence in affecting the attainment of P4 mathematics topics. Moreover, as it was difficult to accurately estimate how much engagement in classroom activities had taken place in the current study, we suggest that future studies provide a better understanding of how teachers set up their classroom environment and whether they adopt pedagogic strategies to engage pupils more thoroughly in mathematics tasks. Accordingly, the short- and longer-term implications of this study can be identified at both the research and policy levels. As mathematics teachers' pedagogic efficacy has been shown to be strongly related to pupils' mathematics achievement, further classroom-based studies are warranted to ascertain what activities, actions and interactions take place in classrooms in which teachers display high levels of pedagogic efficacy (actualising self-reported pedagogic efficacy). These studies should be sensitive to both teacher and pupil behaviour, as one likely explanation for the relationship between pedagogic efficacy and pupil achievement is that teachers and pupils interact at more challenging cognitive levels, with a greater frequency of interaction and with greater sensitivity towards one another when teachers have high versus low levels of such efficacy. Apart from the limitations related to the absence of classroom observations, researchers should be reminded that the measure of PEf in the current study is domain-specific, which is directed to the primary mathematics curriculum in Hong Kong and limited to what those topics the participating teachers doing during the research period. In addition to the specific age-appropriateness of the PEf measure for the P4 level in the current study, it is considered that the results cannot be broadly generalized for teacher professional development in holistic mathematics curriculum or other intellectual territories unless further studies are undertaken with age-appropriate pedagogic topics are developed. With regard to the study's policy implications, we have already noted that high levels of teacher-based mathematics subject knowledge are not associated with high levels of pupil-based mathematics achievement. In conjunction with local and international pressure to move away from a traditional transmission approach towards an inquiry mode of teaching, our results suggest that teachers need to boost their pedagogic efficacy rather than acquire more subject knowledge. The recruitment and training of mathematics teachers, particularly at the primary school level, would be advised to take into account the development and encouragement of pedagogic efficacy in the classroom.

(Word Count: 6957)

Endnotes:

1. Owing to the nature of survey data collection, there were few missing values in our dataset. Using the "mi" package for R, missing values were estimated using multiple imputation (Su, Gelman, Hill, & Yajima, 2011).

2. For those unfamiliar with the relatively new CART technique, it provides a 'data mining approach' that tests for the effects of independent variables on dependent variables without relying on pre-ordained models or hierarchies (for further information, see Hong & Kim, 2008; Ma, 2006; Walker, Lee, & Bryant, 2014).

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Table 1: Descriptive data on schools, teachers and pupils

| Context | | | | Total |
|---|---|---|---|---|
| SCHOOL | | | | |
| Class size | Minimum | Maximum | Average | |
| | 8 | 33 | 22 | |
| TEACHER | | | | |
| Sex | Male | Female | | |
| | 10 (50%) | 10 (50%) | | 20 |
| Mathematics studied to | 16 year exam | 18 year exam | BSc/BEd | |
| | 2 (20%) | 7 (35%) | 11 (55%) | |
| Teaching Experience | 1-5 years | 6-10 years | 11+ years | |
| | 6 (30%) | 6 (30%) | 8 (40%) | |
| SCALES | Mean (SD) | Min/Max | Average score per question | |
| SMK | 31.84 (8.97) | 8/45 | 2.65 | |
| PFC | 49.40 (11.29) | 26/64 | 3.09 | |
| PEf | 64.01 (9.99) | 44/83 | 2.45 | |
| PUPIL | | | | |
| Sex | Male | Female | | |
| | 305 (60.5%) | 199 (39.5%) | | 504 |
| Mathematics achievement | Mean (SD) | Min/Max | | |
| | 72.06 (15.42) | 16/96 | | |

Table 2: Correlations amongst teacher scales and between scales and teacher background

| Scales and aspects | SMK | PFC | PEf | Level of mathematical study | Teaching experience by year |
|---|---|---|---|---|---|
| SMK | 1.00 | | | | |
| PFC | 0.51* | 1.00 | | | |
| PEf | 0.28 | 0.23 | 1.00 | | |
| Level of mathematical study[+] | 0.24 | -0.04 | -0.07 | 1.00 | |
| Teaching experience by year | -0.21 | -0.43 | -0.30 | -0.31 | 1.00 |

*$p < 0.05$

[+]As this measure was ordinal, Spearman's rho correlation was used.

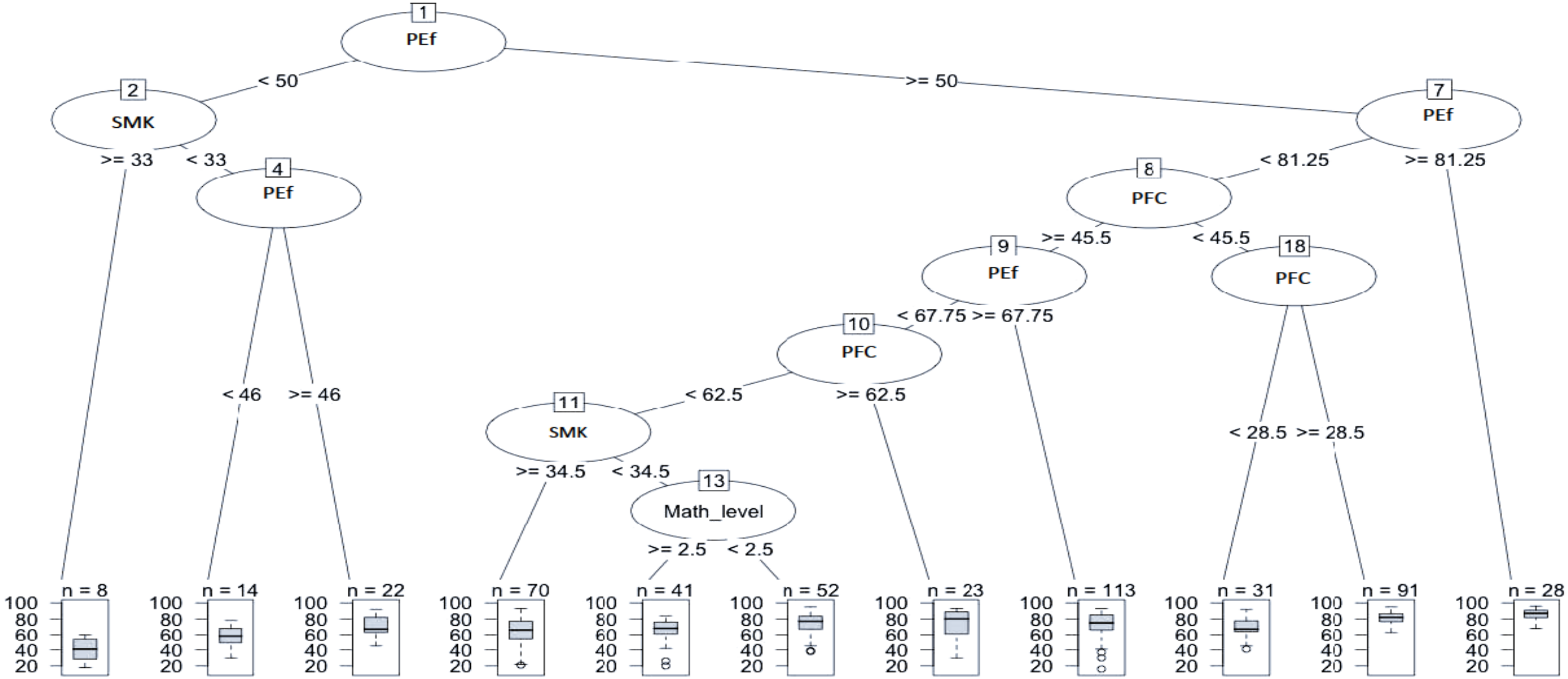
Table 3: Pupil achievement differences by teacher scale scores and background

| Pupil achievement | | SMK[1] | PFC[1] | PEf[1] | Mathematics level[2] | Teaching experience[1] |
|---|---|---|---|---|---|---|
| Mean (SD) | Minimum/maximum scores | | | | | |
| 72.06 (15.42) | 16/96 | NS | NS | t = 5.73** | F (2,492) = 7.25** | t = -3.68** |

[1]Analysis undertaken by linear regression; [2]Analysis undertaken by analysis of variance.

*$p < 0.05$; ** = $p < 0.01$

Figure 1: CART describing teacher factor effects on pupil-/class-level performance



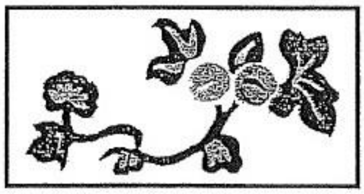
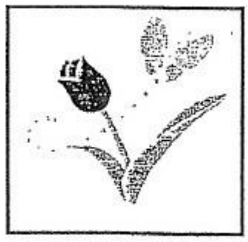

Key: PEf: pedagogic efficacy; SMK: subject matter knowledge; PFC: performance confidence; Math_level: level of mathematics education

**Appendix I:**

Sample questions evaluating pupils' mathematics performance

Attention: Some of the graphics are not drawn in proportion.

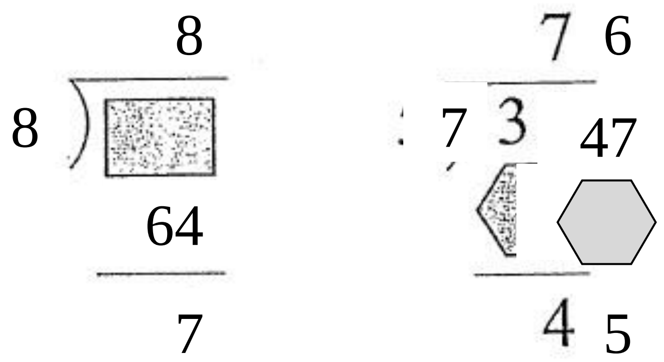
1. Please list the prices of the 3 paintings below from the lowest to the highest.

| A | B | C |
|---|---|---|
| $28 651 | $36 581 | $28 516 |

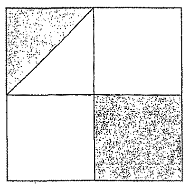
10. One rose costs $6, one chrysanthemum costs $4. Wendy bought one of the flowers and used exactly $102. Which flower did she buy? How many did she buy?

11. If




Then  = ?

15. What is the proportion of the shaded areas in relation to the whole graphic?



**Appendix II:**

Sample questions from Mathematics subject matter knowledge and mathematics efficacy surveys, identifying: a) Level of Mathematics Education (Maths level); b) SMK; c) PFC; and d) PEf

**Maths level**

1. Please state the Grades you have attained in the following subject(s) in the HKCEE and HKALE:

| | | | | | |
|---|---|---|---|---|---|
| (a) | HKCEE | Maths. | ____ | Additional Maths. | ____ |
| (b) | HKALE | A-Level Pure Maths. | ____ | A-Level Applied Maths. | ____ |
| | | AS-Level Maths. & Statistics | ____ | AS –Level Applied Maths. | ____ |

**Sample SMK and PFC questions**

Please write your working and your answers on this Questionnaire. Show or describe your method when the question asks you to.

1. Work out 2915 ÷ 14 without using a calculator. <u>Show your method</u> and give your answer in remainder form.
2. Check that:

$3 + 4 + 5 = 3 \times 4$ $8 + 9 + 10 = 3 \times 9$ $29 + 30 + 31 = 3 \times 30$

Write down a statement (in words) which describes the generalization behind these three examples. Express your generalization using symbolic (algebraic) notation.

Self Audit: <u>Before doing each question</u>, please tick the box that best describes your level of confidence in solving that question.

| | |
|---|---|
| A: | I could explain to someone else how to do this. |
| B: | I feel confident about solving this question. |
| C: | I think I can do this question. |
| D: | I don’t think I can do this / I don’t understand it. |
| E: | Terrifying, I can’t really think about it. |

3. List these decimals in order, from smallest to largest: 0.5, 0.67 and 0.372.

10. Three numbers are written in the bottom row of a pyramid as shown in the figures below. Each number in the rows above (except the bottom row) is the sum of the two numbers directly below it. All the numbers in the pyramid in Figure 1 have been filled in for reference. Find the missing number in the bottom row of the pyramid in Figure 2. Show how you did it.

**Sample PEf questions**

Thinking about **your present class** please **rate** your ability to teach the following topics in mathematics at P4 level (circle the number that best corresponds to how you feel – leave the row blank if your class doesn’t work on this topic)

| | | **1**<br>**very poor**<br>(I hate teaching this - most | **2**<br>**not great**<br>(I don’t look forward to | **3**<br>**OK**<br>(I’m ambivalent – we just do | **4**<br>**good**<br>(I quite enjoy this – most pupils | **5**<br>**excellent**<br>(I love teaching this – pupils |
|---|---|---|---|---|---|---|

| | | | | | | |
|---|---|---|---|---|---|---|
| | | pupils find this difficult and so do I) | this – there are always problems for some) | it and no-one gets that excited) | get it and like working in this) | get it, have fun with it and so do I!) |
| 1 | place value /num. systems | **1** | **2** | **3** | **4** | **5** |
| 2 | addition / subtraction | **1** | **2** | **3** | **4** | **5** |
| 3 | multiplication / division | **1** | **2** | **3** | **4** | **5** |